Dynamics of ghost domains in spin-glasses

2003 J. Phys. A: Math. Gen. 36 10819

(http://iopscience.iop.org/0305-4470/36/43/010)

J. Phys. A: Math. Gen. **36** (2003) 10819–10833 PII: S0305-4470(03)63105-7

Dynamics of ghost domains in spin-glasses

Hajime Yoshino

Department of Earth and Space Science, Faculty of Science, Osaka University, Toyonaka, 560-0043 Osaka, Japan

E-mail: yoshino@ess.sci.osaka-u.ac.jp

Received 6 May 2003, in final form 23 June 2003
Published 15 October 2003
Online at stacks.iop.org/JPhysA/36/10819

Abstract
We revisit the problem of how spin-glasses 'heal' after being exposed to tortuous perturbations by the temperature/bond chaos effects in temperature/bond cycling protocols. Revised scaling arguments suggest that the amplitude of the order parameter within ghost domains recovers very slowly, compared with the rate it is reduced by the strong perturbations. The parallel evolution of the order parameter and the size of the ghost domains can be examined in simulations and experiments by measurements of a memory autocorrelation function which exhibits a 'memory peak' at the timescale of the age imprinted in the ghost domains. These expectations are confirmed by Monte Carlo simulations of an Edwards–Anderson Ising spin-glass model.

PACS numbers: 74.60.Ge, 02.50.Ey, 75.50.Lk

(Some figures in this article are in colour only in the electronic version)

1. Introduction

A class of scaling theories [1–3] for randomly frustrated glassy systems has pointed out a striking fragility of their free-energy landscapes. While they realize some glassy order within a given environment specified for instance by temperature, even an infinitesimal change of the latter leads to radical reformation of the free-energy landscape to a globally uncorrelated new one. Such non-perturbative, global shuffling of the free-energy landscape with infinitesimal changes of control parameters are called *chaos effects*. Indeed, theoretical studies of some microscopic models, including studies on Edwards–Anderson (EA) Ising spin-glass models by the Migdal–Kadanoff renormzalization group (MKRG) method [4–6] and mean-field theory [7] (and references therein) and directed polymers in random media (DPRM) [8], have partially or almost fully confirmed such striking effects. Further works may clarify to what extent these unusual phenomena are universal.

0305-4470/03/4310819+15$30.00

It is naturally interesting to see how slowly relaxing or ageing glassy systems will react to such tortuous perturbations [9–14]. While systems such as simple phase separating systems would either keep ageing accumulatively (domain growth) or stop ageing under *external* driving forces (e.g. stirring oil+vinegar) [15], spin-glasses exhibit rejuvenation-memory effects [9] which are far more puzzling and richer. In [12] a minimal description for such a dynamics was obtained for the case of Ising spin-glasses in terms of *ghost domains*, which is a direct extension of the concept of the standard scaling theory for domain growth [16] in isothermal ageing. In contrast to isothermal ageing, the amplitudes of the order parameters or bias within domains become dynamical variables which play a central role; they act as *internal* driving forces which perturb the trajectory of the domain growth itself. As the result, a concrete mechanism of imprinting/retrieving multiple memory under the tortuous chaos effects was found. Recently, the MKRG method was applied to the dynamics of the EA model subjected to chaos effects and such a mechanism was demonstrated explicitly [13].

In this paper we revise the ghost domain scenario based on the theory by Bray and Kisner [17] on the growth of the bias during domain growth dynamics. We consider a simple one-step 'perturbation–healing' protocol. An example is the one-step temperature-cycling protocol [18, 19] first used in spin-glasses. It proceeds as follows.

(1) *Initial ageing stage*. First a spin-glass is equilibrated at a high enough temperature above the glass transition temperature T_g. Then, at time $t = 0$, the temperature is quenched down to a temperature, say T_A, below T_g where the system is aged for some time t_w. This stage is just the same as usual isothermal ageing.

(2) *Perturbation stage*. The temperature is changed to $T_B = T_A + \Delta T$ (with ΔT being either positive or negative) where the system is aged for some time τ_p. Strong restart of ageing or *rejuvenation* is observed, for instance, by measuring the ac magnetic susceptibility in the spin-glasses and ceramic superconductors [20, 21]. Other glassy systems, such as super-cooled liquids [22] and polymer glasses [23], exhibit no or much weaker rejuvenations. This may suggest the absence of chaos effects in some classes of glassy systems. We should also bear in mind that large enough length/timescales compared with the overlap length (see equation (3)) must be explored to see chaos effects. Failures of some experiments and simulations to detect rejuvenations may be related to this difficulty.

(3) *Healing stage*. Finally the temperature is put back to T_A. In spin-glasses, strong restart of ageing or rejuvenation is observed again [11, 19, 24]. After some *recovery time* say τ_{rec} this restarted process disappears and the rest of the relaxation becomes a continuation of the initial ageing stage, which is called the *memory effect*. We will closely discuss the two-stage processes in the healing stage based on the ghost domain scenario.

Many systems 'heal' by waiting some recovery time τ_{rec} after being exposed to a perturbation for a certain time τ_p. Simple minded 'length scale(s)' (or some equivalent 'energy-barrier') arguments which neglect the internal driving due to the remanent bias may lead to two contradictory possibilities: (a) healing is *impossible* after such strong perturbations due to chaos effects or (b) healing is *somehow possible* and the recovery time τ_{rec} is just identical to the timescale at which the length scale $L(\tau_{\mathrm{rec}})$ (energy barrier) explored after switching off the perturbation becomes as large as the length scale $L(\tau_p)$ (energy barrier) explored during the perturbation. Furthermore, we could argue that the 'effective age' of the system imprinted in the system would be largely modified once $L(\tau_p)$ becomes larger than the length (energy) scale corresponding to the age. Somewhat surprisingly, we find that all these intuitions fail in general for the perturbations operated in the strongly perturbed regime of the chaos effect. In this paper we also consider dynamics operated in the weakly perturbed regime of the chaos effect. This allows us to take into account the effects of slowness of the switching on/off perturbations in realistic circumstances.

After introducing the spin-glass model in the next section, the definition of ghost domains is summarized and the scaling theory by Bray and Kisner is briefly reviewed in section 3. In sections 4 and 5 the revised ghost domain scenario is introduced focusing on the simple one-step perturbation–healing protocol mentioned above and the scenario is examined numerically on the four-dimensional EA Ising spin-glass model. In section 6 we propose a simple way to take into account the effects of slow switchings such as heating/cooling rate effects. The conclusion of the paper is presented in the last section.

2. Model

Specifically we consider the EA Ising spin-glass models described by a Hamiltonian

$$H = -\sum_{i,j} J_{ij} S_i S_j \tag{1}$$

where S_i is an Ising spin at site i located at $\vec{r}_i$ on a d-dimensional lattice with N lattice sites, and J_{ij} is a random interaction bond which takes $+J$ and $-J$ randomly for each nearest neighbour pair (i, j). Here, $J > 0$ is the unit of energy scale. For convenience, we denote the scaled thermal energy $k_B T/J$ as temperature T in the following. Here k_B is the Boltzmann constant. We consider two types of perturbation: (1) *temperature changes* $T \to T + \Delta T$; (2) *bond changes* $\mathcal{J} \to \mathcal{J}'$. A new set of bonds $\mathcal{J}' = \{J'_{ij}\}$ is created from the original one $\mathcal{J} = \{J_{ij}\}$ as follows. For each pair (i, j) we choose $J'_{ij} = -J_{ij}$ randomly with probability p and $J'_{ij} = J_{ij}$ with probability $1 - p$.

In the numerical simulation presented in section 5 we use the $d = 4$ model on the hypercubic lattice and the single spin flip heat-bath Monte Carlo method. In simulations we limit ourselves to bond changes since computational power is much too limited to study temperature changes efficiently.

3. Ghost domains

Let us introduce the basic ingredients of the ghost domain scenario to prepare for the discussion of the simple one-step perturbation–healing protocol (e.g. the one-step temperature-cycling experiments) in the next two sections. For simplicity we assume that an equilibrium states $\Gamma^{T,\mathcal{J}}$ of a spin-glass system with a set of random interaction bonds $\mathcal{J}$ at temperature T below the spin-glass transition temperature T_g is given by its *typical* spin configuration. Such a configuration at site i may be described as $\sqrt{q_{\mathrm{EA}}}(T)\sigma_i^{T,\mathcal{J}}(t)$ where $\sigma_i^{T,\mathcal{J}}(t)$ is an Ising variable and $q_{\mathrm{EA}}(T)$ is the EA order parameter which takes into account the effects of thermal fluctuations. Furthermore, we assume that the only possible other phase at the same environment $(T, \mathcal{J})$ is $\bar{\Gamma}^{T,\mathcal{J}}$ whose configuration is given by $-\sqrt{q_{\mathrm{EA}}}(T)\sigma_i^{T,\mathcal{J}}$. However, extensions to the cases that more phases exist for a given environment may be considered as well.

3.1. Weakly and strongly perturbed regimes of chaos effects

The chaos effects become stronger at larger length scales. Since the distinctions between the weakly and strongly perturbed regimes are important in the following, here we summarize the picture on the crossover between the two regimes given in [8, 13, 25].

Let us consider a generic perturbation which may induce a droplet excitation of size L with respect to the 'ground state' $\{\sigma_i^{T,\mathcal{J}}\}$. The excited state has a certain free-energy gap $F_L(>0)$ with respect to the ground state. Suppose that we have a perturbation such that the excited state obtains a *gain* of the free-energy of order $\Delta U_L/J = \delta(L/L_0)^a$ where L_0 is a microscopic unit length scale. Then a droplet excitation will be induced if ΔU_L turns out to be greater than the free-energy gap F_L. The free-energy gap is expected to have a broad distribution characterized by a distribution function $\rho_L(F_L)$ with the scaling form [1, 2], $\rho_L(F_L)\,\mathrm{d}F_L = \tilde{\rho}(F_L/J(L/L_0)^\theta)\,\mathrm{d}F_L/J(L/L_0)^\theta$ where $J(L/L_0)^\theta$ is the typical free-energy gap with θ (>0) being the stiffness exponent. Using these properties the probability $p_L(\delta)$ that a perturbation of strength δ induces a droplet excitation of size L is found as

$$p_L(\delta) \sim \int_0^{\Delta U_L} \mathrm{d}F_L \rho(F_L) = \int_0^{(L/\xi(\delta))^\zeta} \mathrm{d}y \tilde{\rho}(y) \tag{2}$$

where

$$\xi(\delta) = L_0 \delta^{-1/\zeta} \tag{3}$$

is the characteristic crossover length, called the overlap length, beyond which $p_L(\delta)$ becomes $O(1)$. The exponent ζ, the so-called chaos exponent, is given by $\zeta = a - \theta$. We can see that if $\zeta > 0$ ($a > \theta$) the probability $p_L(\delta)$ continuously increases with increasing $L/\xi(\delta)$. In the following, we distinguish between the *strongly perturbed regime* $L/\xi(\delta) > 1$ and the *weakly perturbed regime* $L/\xi(\delta) < 1$.

In the *strongly perturbed regime* $L/\xi(\delta) > 1$, the original ground state $\left\{\sigma_i^{T,\mathcal{J}}\right\}$ is completely unstable with respect to the droplet excitations, i.e. a new equilibrium state must form. The term *chaos* [1–3, 26] properly describes the fact that a strongly perturbed regime eventually emerges even for arbitrary small $\delta \ll 1$ at sufficiently large length scales. However, chaos does not set in abruptly at the overlap length $\xi(\delta)$ but in the *weakly perturbed regime* $L/\xi(\delta) < 1$ chaos-like droplet excitations already occur at length scales smaller than $\xi(\delta)$ with non-zero probability $p_L(\delta)$ [8, 13, 25].

In the case of temperature shifts of strength ΔT the possible free-energy gain of a droplet excitation of size L is the entropy gain ($\times\Delta T$) which is expected to scale as $\Delta U_L/J \sim \Delta T(L/L_0)^{d_s/2}$ where d_s is the surface fractal dimension of droplet excitations (see [6] for a detailed discussion). In the case of bond perturbations, the random gain of energy of a droplet excitation occurs at around its surface so that $\Delta U_L/J \sim p(L/L_0)^{d_s/2}$. Thus, temperature and bond perturbations should lead to a chaos effect of the same universality class with $\zeta = d_s/2 - \theta$.

3.2. Definition of ghost domains

Let us consider a generic protocol such that the working environment is changed from time to time among a set of target environments $\{A, B, \ldots\}$ which consists of different temperatures $\{T_A, T_B, \ldots\}$ (all below T_g) and/or different bonds $\{\mathcal{J}_A, \mathcal{J}_B, \ldots\}$ whose equilibrium states are represented by $\sqrt{q_{\mathrm{EA}}(T_A)}\sigma_i^A, \sqrt{q_{\mathrm{EA}}(T_B)}\sigma_i^B, \ldots$.

Suppose that the system is now evolving in a certain working environment, say $W = (T_W, \mathcal{J}_W)$ at a certain time t. Short time averages may be taken to average out short time thermal fluctuations. Then the temporal spin configuration can be represented as $\sqrt{q_{\mathrm{EA}}(T_W)}s_i(t)$ where $s_i(t)$ takes Ising values. It can be projected on to the equilibrium states of *any* environment $R \in \{A, B, \ldots\}$ as

$$\tilde{s}_i^R(t) = \sigma_i^R s_i(t). \tag{4}$$

Then the projected image $\tilde{s}_i^R(t)$ is described in a coarse-grained way by the following two features.

(i) the *domain wall configuration*: the configuration of the spatial pattern of the *sign* of the projection $\tilde{s}_i^R(t)$;

(ii) the *order parameter*: the amplitude of the projection $\rho^R(t) = \left|\left[\tilde{s}_i^R(t)\right]_{\text{domain}}\right|$ where $[\cdots]_{\text{domain}}$ denote the spatial average within a ghost domain.

It is useful to consider decomposition of a ghost domain Γ^R ($\bar{\Gamma}^R$) into 'patches':

- the strength of the bias has the full amplitude 1 within a patch;
- the 'signs $+/-$' of the bias are however different on different patches–the majority has the same sign as that of the ghost domain to which they belong to, and minorities have the opposite sign.

The probability $p_{\text{minor}}(t)$ that a patch belongs to the *minority phase* $\bar{\Gamma}^R$ (Γ^R) in a ghost domain of Γ^R ($\bar{\Gamma}^R$) is related to the strength of the bias $\rho^R(t)$ as

$$p_{\text{minor}}(t) = (1 - \rho^R(t))/2. \tag{5}$$

If we choose $R = W$, a ghost domain reduces to an ordinary domain which is enough in isothermal ageing where the order parameter is a constant. In the cycling protocols, *minimal* description is to keep track of projections on to the equilibrium states of *all* target environments. We call such projections *ghost domains*. It is very important that not only (i) the domain wall structures but also (ii) the amplitude of the order parameter within the domains are dynamical variables.

3.3. Physical observable

Let us note here that the basic quantities measured in experiments and simulations are essentially *gauge-invariant* (or *ghost-invariant*), i.e. they do not depend on specific choice of projections. An important example is the spin auto-correlation function

$$C(t, t') = (1/N) \sum_i \langle S_i(t) S_i(t') \rangle \tag{6}$$

where N is the number of spins and $\langle \cdots \rangle$ represents taking an average over different trajectories. The auto-correlation function can be re-expressed in terms of any projection field $\tilde{s}_i^R(t)$ as $C(t, t') = q_{\text{EA}}(T_R)(1/N) \sum_i \left\langle \tilde{s}_i^R(t) \tilde{s}_i^R(t') \right\rangle$ because $\sigma_i^R = \pm 1$. Thus, it does not depend explicitly on the specific choice of projections, i.e. gauge-invariant except for the prepfactor. The auto-correlation function can be measured experimentally by monitoring spontaneous thermal fluctuations of the magnetization $M(t)$ [27] because the leading $O(N)$ part of the magnetic auto-correlation function is $NC(t, t')$ in spin-glasses with no ferromagnetic or anti-ferromagnetic bias in the distribution of the bonds. In experiments, linear magnetic susceptibilities to uniform external magnetic field are often measured. By the same token as above, it can be seen that the linear magnetic response functions (per spin) of spin-glasses to a uniform external field $h(t')$, $R(t, t') = (1/N)\partial \langle M(t) \rangle / \partial h(t')$ are essentially gauge-invariant.

3.4. Basic dynamics at a working environment

Suppose that the system is temporally evolving at a certain working environment W with temperature $T = T_W$ with a certain set of bonds $\mathcal{J}_W$. Here we summarize the basic properties of the dynamics of the (ghost) domains of $\Gamma^W/\bar{\Gamma}^W$.

At the coarse-grained mesoscopic level, the relaxational dynamics is considered as a thermally activated process of a droplet-like excitation. The energy barrier associated with a droplet of size L scales as $E_b \sim \Delta(T)(L/L_0)^\psi$ with $\psi > 0$. Thus, at a given logarithmic timescale $\log(t/\tau_0(T))$ a droplet as large as

$$L_T(t) \sim L_0[(k_B T/\Delta(T)) \ln(t/\tau_0(T))]^{1/\psi} \tag{7}$$

can be thermally activated [26]. In the above formula the effects of critical fluctuations can be taken into account in a renormalized way in the characteristic energy scale $\Delta(T)$ for the free-energy barrier and the characteristic timescale $\tau_0(T)$.

Suppose that the projection of the initial spin configuration on to the equilibrium state Γ^W at time $t = 0$ is strongly disordered such that its spatial correlation function decays rapidly beyond some correlation length $\xi_{\rm ini}$:

$$\left[\left(\tilde{s}_i^W(0) - \rho^W(0)\right)\left(\tilde{s}_j^W(0) - \rho^W(0)\right)\right] = F\left(\frac{|(\vec{r}_i - \vec{r}_j)|}{\xi_{\rm ini}}\right) \qquad \left[\tilde{s}_i^W(0)\right] = \rho^W(0). \tag{8}$$

Here $[\cdots]$ denotes the average over space and $F(x)$ is a certain rapidly decreasing function. Note that the bias $\rho^W(0)$ is made *homogeneous* within the system.

Domain growth without bias. If the initial bias is absent $\rho^W(0) = 0$, the global Z_2 symmetry of the system is not broken and the domain growth (ageing) never stops. The mean separation between the domain walls at time t is $L_T(t)$ given in equation (7) [26]. In such a *critical quench* the spatial correlation function

$$C^W(r, t, t') = \left[\left\langle \tilde{s}_i^W(t)\tilde{s}_j^W(t')\right\rangle\right]_{r=|\vec{r}_i - \vec{r}_j|} \tag{9}$$

exhibits universal scaling properties [16]. In the so-called ageing regime $L_T(t) > L_T(t')$ it scales as

$$C_0^W(r, t, t') \sim \left(\frac{L_T(t)}{L_T(t')}\right)^{-\bar{\lambda}} h\left(\frac{r}{L_T(t')}\right) \qquad L_T(t) > L_T(t'). \tag{10}$$

The subscript '0' is meant to emphasize that the spin configuration is random at $t = 0$ with respect to the target equilibrium state. Here, $h(x)$ is a decreasing function with $h(0) = 1$. The exponent $\bar{\lambda}$ is a non-equilibrium dynamical exponent introduced by Fisher and Huse [26]. (Note that in some studies, e.g. [26], and also [12, 28], $\bar{\lambda}$ is denoted as λ. In the present paper we follow [16, 17] and use $\bar{\lambda}$ for the decay of the correlation function and λ for the growth of bias discussed below (see equation (12)).)

A special case of much interest is the auto-correlation function ($r = 0$) which is just the spin auto-correlation function $C(t, t')$ defined in equation (6) which is a gauge-invariant quantity. It generically follows a scaling of the form $C_0(t, t') = C_0(L_T(t)/L_T(t'))$. The scaling function $C_0(x)$ remains at 1 in the quasi-equilibrium regime $x < 1$. In the ageing regime $x > 1$, it approaches 0 asymptotically as $C_0(x) \sim x^{-\bar{\lambda}}$. In Ising spin glasses

$$d/2 \leqslant \bar{\lambda} < d \tag{11}$$

is proposed [26]. This very slow relaxation is in sharp contrast to the exponential decay in the paramagnetic phase $C_0(t, t') \propto \exp(-|t - t'|/\tau_{\rm eq}(T))$ where $\tau_{\rm eq}(T)$ is the correlation time in the paramagnetic phase.

Domain growth with bias. Even if the initial bias $\rho^W(0)$ is small, the Z_2 symmetry is explicitly broken if it is non-zero. We expect that the strength of the symmetry breaking will increase with time and eventually terminate the ageing just as if *external* symmetry breaking field is applied. This problem was considered theoretically first by Bray and Kisner [17]. They noticed that the non-zero *homogeneous* bias grows with time t as

$$\rho^W(t) \sim \rho^W(0)\left(\frac{L_T(t)}{\xi_{\rm ini}}\right)^{\lambda} \tag{12}$$

and the dynamical exponent λ is related to $\bar{\lambda}$ as

$$\bar{\lambda} + \lambda = d. \tag{13}$$

Here let us summarize the derivation [17] within our context. First, we can see that the bias is nothing but the $k = 0$ component of the Fourier transform of $\tilde{s}_i^W$. Then we assume that the amplitude of the $k = 0$ component at time t can be computed as a linear response to the change of its initial value. Secondly, assuming the Gaussian characteristics of the random initial condition equation (8) we find that the linear response function is the same as the $k = 0$ component of the spin correlation function equation (9) up to some proportionality constant c. As a result we obtain

$$\rho^W(t) = c\rho^W(0)C_{k=0}(t, 0). \tag{14}$$

Then, using equation (9) we find equations (12) and (13). Combining the scaling relation equation (13) and the inequality equation (11) we find

$$\bar{\lambda}/\lambda \geqslant 1. \tag{15}$$

As we discuss below, this inequality suggests that the healing of spin-glasses after chaotic perturbations takes an enormously long time[1].

4. A cycle on a globally symmetry broken state

Let us now begin with the perturbation–healing protocol by considering an idealized limit. This corresponds, for example, to the one-step temperature-cycling experiments mentioned in the introduction $T_A \to T_B \to T_A$ but with the initial ageing done for an extremely long time $t_w = \infty$; the spin configuration is globally equilibrated with respect to an equilibrium state Γ^A at the beginning. Then *perturbation* is performed; we change the temperature or bond and let the system evolve for a certain time τ_p so that domains of $\Gamma^B/\bar{\Gamma}^B$ grow. Lastly *healing* is performed; we switch off the perturbation and let the system evolve afterwards for some time τ_h. Here A and B denote (1) T_A and $T_B = T_A + \Delta T$ in the case of temperature cycling or (2) $\mathcal{J}_A$ and $\mathcal{J}_B$ (which is created by randomly changing the sign of a fraction p of the bonds in the original set $\mathcal{J}_A$) in the case of bond cycling. For simplicity, we assume the switchings are instantaneous. The effects of slow switching times, e.g. finite heating/cooling rates, will be discussed later in section 6.

Strongly perturbed regime. If the duration of the perturbation τ_p is long enough such that $L_B(\tau_p) > \xi(\delta)$, where $\xi(\delta)$ is the overlap length given in equation (3) and δ can be either ΔT or p, the strongly perturbed regime of the chaos effects (see section 3.1) should come into play. For simplicity, here we neglect dynamics at short timescales which belong to the weakly perturbed regime. An extreme example of $\xi(\delta) = 1$ is shown in figure 1 using a Monte Carlo simulation of a two-dimensional Ising Mattis model.

The initial spin configuration is globally aligned to Γ^A so that it is fully biased as $\tilde{s}_i^A = 1$. But simultaneously $\tilde{s}_i^B$ is completely disordered (beyond $\xi(\delta)$) with no bias $\left[\left\langle \tilde{s}_i^A \right\rangle\right] = 0$. Thus, during the perturbation stage, the domains of both $\Gamma^B/\bar{\Gamma}^B$ grow, competing with each other just as the usual domain growth. Their typical size becomes $L_B(\tau_p)$. As can be seen in figure 1, this amounts to reduction of the bias (or staggered magnetization) with respect to Γ^A. The remanent bias ρ_{rem}^A becomes

$$\begin{aligned}\rho_{\text{rem}}^A(\tau_p) &= (1/N)\sum_i \tilde{s}_i^A(\tau_p) = (1/N)\sum_i \tilde{s}_i^A(0)\tilde{s}_i^A(\tau_p) = (1/N)\sum_i \tilde{s}_i^B(0)\tilde{s}_i^B(\tau_p) \\ &= C_0(\tau_p, 0) \sim (L_B(\tau_p)/\xi(\delta))^{-\bar{\lambda}}. \end{aligned}\tag{16}$$

[1] Unfortunately the spherical model studied in [12] is special in the sense that $\bar{\lambda} = \lambda = d/2$ so that $\bar{\lambda}/\lambda = 1$. In the latter study, the possibility of the enormously large recovery time was not noticed.

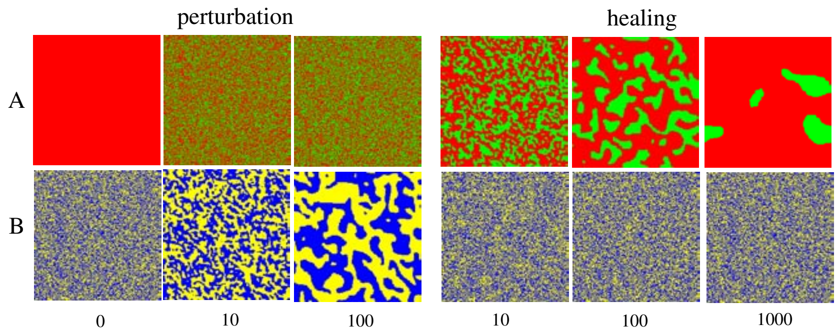

Figure 1. Evolution of ghost domains in a simple 'perturbation–healing' protocol on a globally symmetry broken state. This is a demonstration using a heat-bath Monte Carlo simulation of a two-dimensional Ising Mattis model ($N = 400 \times 400$) in which the interaction bonds in equation (1) are given as $J_{ij} = J\sigma_i\sigma_j$ where σ_i is a random Ising (gauge) variable given at each site. We immediately find that the equilibrium state (ground state) for each $\mathcal{J}$ is simply given by the set $\{\sigma_i\}$. The initial spin configuration is chosen to be identical to a random ground state σ^A. *Perturbation*: for time $\tau_p = 100$ (MCS) the system is strongly perturbed by using the Hamiltonian of a different ground state σ^B which is completely uncorrelated with σ^A: $\xi(\delta) = 1$ in the unit lattice. *Healing*: then the Hamiltonian is put back to the original one which is used for additional 1000 (MCS). In the present examples, the temperature is set to $T = 2.0$. The different colours (greyscales) represent the signs + and − of the projections on to the ground states. The three columns on the left sides are snapshots at time 0, 10, 100 (MCS) during the perturbation. Domains of $\Gamma^B/\bar{\Gamma}^B$ grow while the bias ρ^A decreases. In this example, ρ^A has become 0.03 which is too small to distinguish by eye. In the three columns on the right are snapshots at time 10, 100, 1000 (MCS) during the healing. Here domains of $\Gamma^A/\bar{\Gamma}^A$ grow but the minority phase $\bar{\Gamma}^A$ slowly disappears and ρ^A increases. The recovery of ρ^A turns out to be a much longer time than τ_p. In this model the growth law equation (7) should be replaced by $L(t)/L_0 \sim \sqrt{t/t_0}$ since it is equivalent to the ferromagnetic Ising model [16]. We found numerically $\lambda \sim 0.8$ and $\bar{\lambda} \sim 1.2$ in this model being consistent with equation (13).

Here we have used the properties of the initial condition and the gauge invariance of the auto-correlation function. The subscript '0' is used in the last equation because the initial condition is completely random with respect to the relaxational dynamics which is *at work* during the perturbation. Here it can be seen clearly that the strong perturbation due to the chaos effect is *not* equivalent to put a system into a paramagnetic phase. In the latter case, we would find exponential decay of the bias $\rho^A_{\mathrm{rem}}(\tau_p) \sim \exp(-\tau_p/\tau_{\mathrm{eq}}(T_B))$ where $\tau_{\mathrm{eq}}(T_B)$ is the relaxation time in the paramagnetic phase. Thus, the chaos effect does *not* amount to pushing the system to the disordered phase, contrary to what might have been suspected.

During the healing stage, the domains of both $\Gamma^A/\bar{\Gamma}^A$ grow competing with each other starting from the minimum length scale $\xi(\delta)$. Their typical size becomes $L_A(\tau_h)$. However, this is a domain growth with biased initial condition as equation (8). From equation (12) the strength of the global bias is found to grow as

$$\rho^A_{\mathrm{rec}}(\tau_h) = \rho^A_{\mathrm{rem}}(L_A(\tau_h)/\xi(\delta))^{\lambda} \tag{17}$$

where ρ^A_{rem} is the remanent bias at the end of the perturbation stage (or the beginning of the healing stage). Note that $\rho^A_{\mathrm{rec}}(\tau_h)$ is *proportional to* the initial remanent bias ρ^A_{rem} which is the direct consequence of the 'linear-response' of the temporal bias with respect to the change of the initial bias as noticed by Bray and Kisner.

The above situations may be rephrased as follows. During the perturbation stage, the system may be decomposed into patches whose size is given by the overlap length $\xi(\delta)$. The probability p_{minor} that a patch belongs to the *minority phase* $\bar{\Gamma}^A$ is related to the bias ρ^A

as given in equation (5) so that it increases with τ_p during the perturbation stage. Next, in the healing stage the system may be decomposed into patches of size $L_A(\tau_h)$, which now grows with τ_h. The probability p_{minor} now decreases because ρ^A increases. Consequently, the minority phase eventually disappears and the domain growth stops at the recovery time τ_{rec}.

Combining equations (16) and (17) we find the recovery time $\tau_{\mathrm{rec}}^{\mathrm{strong}}$ of the strength of the bias as

$$\frac{L_A\left(\tau_{\mathrm{rec}}^{\mathrm{strong}}\right)}{\xi(\delta)} = \left(\frac{L_B(\tau_p)}{\xi(\delta)}\right)^{\bar{\lambda}/\lambda}. \tag{18}$$

Here the superscript 'strong' is meant to emphasize that it is a formula valid in the strongly perturbed regime. Since $\bar{\lambda}/\lambda \geqslant 1$ as given in equation (15), we conclude that the recovery time can be significantly large (see footnote 1).

The above considerations can be extended to multistep cycling. Very counterintuitive consequences follow due to the multiplicative nature of the effect of multiple perturbations [12]. For example, another perturbation stage to grow $\Gamma^C/\bar{\Gamma}^C$ for some time τ_p' can be added before the healing stage in the perturbation–healing protocol discussed above. Here Γ^C is assumed to be decorrelated with respect to both Γ^A and Γ^B beyond the overlap length $\xi(\delta)$. Then the recovery time $\tau_{\mathrm{rec}}^{\mathrm{strong}}$ of the order parameter of A becomes

$$\frac{L_{T_A}\left(\tau_{\mathrm{rec}}^{\mathrm{strong}}\right)}{\xi(\delta)} = \left(\frac{L_B(\tau_p)}{\xi(\delta)}\frac{L_C(\tau_p')}{\xi(\delta)}\right)^{\bar{\lambda}/\lambda} \tag{19}$$

which can yield huge recovery time.

Weakly perturbed regime. If the duration of the perturbation τ_p is small such that $L_B < \xi(\delta)$, the effect of the perturbation should remain mild, as explained in section 3.1. Here the mutual interferences between ghost domains just amount to induce *rare* droplet excitations of various size L with probability $p_L(\delta) \ll 1$ (see equation (2)) on top of each other. They are just isolated islands of the minority phase which rarely overlap with each other. Thus, we only need to keep track of switch on/off of such independent objects during perturbation and healing stages. This means a naive 'length scale(s)' argument to estimate the recovery time of bias fortunately does not fail in this regime.

More precisely, the result of [13] implies the remanent bias decreases due to the increase of rare islands of the minority phase as

$$\rho_{\mathrm{rem}}^A(\tau_p) = 1 - c p_{L_B(\tau_p)}(\delta) + O(p^2) \simeq 1 - c(L_B(\tau_p)/\xi(\delta))^{\zeta} \tag{20}$$

in the perturbation stage and increases as

$$\rho_{\mathrm{rec}}^A(\tau_h) = \rho_{\mathrm{rem}}(\tau_p) + c(L_A(\tau_h)/\xi(\delta))^{\zeta} \tag{21}$$

by removing islands one by one in the healing stage. Here c is some numerical constant. We have neglected higher-order terms of $O(p^2)$. In the MKRG analysis [13], it has been shown that both equations (20) and (16) are limiting behaviours of a universal scaling function of $L_B/\xi(\delta)$. Note that the bias recovers in *additive* fashion in equation (21) which is markedly different from the *multiplicative* fashion found in the strongly perturbed regime equation (17).

We find that the recovery time in the weakly perturbed regime is simply given as

$$L_A\left(\tau_{\mathrm{rec}}^{\mathrm{weak}}\right) = L_B(\tau_p) \qquad \text{or} \qquad \tau_{\mathrm{rec}}^{\mathrm{weak}}\big/\tau_0(T_A) = (t/\tau_0(T_B))^{(\Delta(T_A)/\Delta(T_B))(T_B/T_A)}. \tag{22}$$

Here the superscript 'weak' is meant to emphasize that it is a formula valid only in the weakly perturbed regime. The second equation holds in the case of activated dynamics equation (7) which simplifies further at low enough temperature as $\tau_{\mathrm{rec}}^{\mathrm{weak}}\big/\tau_0 = (t/\tau_0)^{(T_B/T_A)}$ where temperature dependence of the unit time $\tau_0(T)$ and the energy scale $\Delta(T)$ can be neglected.

There, we only need to know the microscopic timescale τ_0, which is known to be around 10^{-12}–10^{-13} s in real spin-glass materials, to estimate the recovery time $\tau_{\mathrm{rec}}^{\mathrm{weak}}$.

Moreover, it is easy to see that non-overlapping islands of the minority phase cannot cause any non-trivial effect of multiple perturbations (see equation (19)). This point becomes important when we consider the effects of slow switching, e.g. heating/cooling rate effects in section 6.

Other non-chaotic, *mild* effects of perturbations can be considered in similar ways. For instance, the change of thermally active droplets can be taken into account by changing $p_L(\delta)$ above by $\Delta T/(L/L_0)^{\theta}$. The latter amounts to an even weaker effect at larger timescales but may be dominant at short timescales.

5. Parallel evolution of domain sizes and order parameter

Let us complete the scenario for the one-step perturbation–healing protocol by now allowing the waiting time t_w in the initial ageing to be *finite*. Suppose the system is completely disordered with respect to both Γ^A and Γ^B at the beginning and aged for some waiting time t_w at A. Then, instead of an infinitely large domain of Γ^A, there will be domains of Γ^A and $\bar{\Gamma}^A$ of size $L_A(t_w)$. In the following, we consider the perturbation–healing protocol exerted on to this initial state.

The time evolution of the system in a cycling protocol can be concisely described by the time evolution of the ghost domains of $\Gamma^A/\bar{\Gamma}^A$ and $\Gamma^B/\bar{\Gamma}^B$ [12]. The situation may be visualized again simply by patches. A ghost domain Γ^A of size $L_A(t_w)$ may be decomposed into patches of size $\xi(\delta)$ during the perturbation stage and patches of size $L_A(\tau_h)$ during healing stage. Within a patch the bias is always homogeneous and has the full amplitude 1. But the 'signs' of the bias is different on different patches. The probability p_{minor} (equation (5)) that a patch belong to the *minority phase* $\bar{\Gamma}^A$ increases in time as equation (16) in the perturbation stage and decreases with time as equation (17) in the healing stage. Since the size of the ghost domain itself is finite, it also continues to grow during the healing stage. Following [12] we call the new domain growth under the background bias field during the healing *inner-coarsening* and the further growth of the size of the ghost domain itself *outer-coarsening*.

The crucial point is that the projection $\tilde{s}^A$ keeps the same long wavelength spatial structure of *sign* of the bias as the original domain structure just before the perturbation throughout the perturbation–healing stages. In the absence of such an explicit mechanism of a sort of symmetry breaking, the new domains grown during the healing would have completely wrong signs of bias and could lead to total erasure of the memory (the scenario (a) mentioned in the introduction). The latter was the main problem in the previous attempt to model multiple domains by Koper and Hilhorst [29] and many other popular 'length scale(s)' arguments which neglect the role of the internal driving by the remanent bias.

5.1. Memory correlation function

The memory of the 'state' of the system just before the perturbation (or the end of the initial ageing stage) can be directly quantified by the spin auto-correlation function as

$$C_{\mathrm{mem}}(\tau + t_w, t_w) = C(\tau + t_w, t_w), \tag{23}$$

which we call the *memory correlation function*. The hamming distance $d = 1 - C_{\mathrm{mem}}$ gives a measure of the closeness in the phase space between the phase points at the two times. Note that in the limit $t_w \to \infty$ it reduces to the global bias ρ^A discussed in section 4. It is useful to recall that the auto-correlation function is gauge-invariant so that it is suitable for experiments/simulations of spin-glass systems where we do not know *a priori* any equilibrium states below T_g.

Strongly perturbed regime. Let us first consider a cycling operated in the strongly perturbed regime. During the perturbation stage we can easily see that C_{mem} is identical to $\sqrt{q_{\mathrm{EA}}(T_A)}\sqrt{q_{\mathrm{EA}}(T_B)}\rho^A_{\mathrm{rem}}(\tau_p)$ where $\rho^A_{\mathrm{rem}}(\tau_p) = C_0(\tau_p, 0) \sim (L_B(\tau_p)/\xi(\delta))^{-\bar{\lambda}}$ (see equation (16)). During the healing stage the analytical result of a spherical Mattis model suggests the following factorization (see equation (109) of [12])

$$C_{\mathrm{mem}}(\tau_h + \tau_p + t_w, t_w) = q_{\mathrm{EA}}(T_A)\rho^A_{\mathrm{rec}}\big(\tau_h; \rho^A_{\mathrm{rem}}\big)C_0(\tau_h + t_w, t_w). \tag{24}$$

Here the factor $\rho^A_{\mathrm{rec}}\big(\tau_h; \rho^A_{\mathrm{rem}}\big)$ represents the growth of the bias within the ghost domains by the *inner-coarsening* (see equation (17)) and the factor $C_0(\tau_h + t_w, t_w)$ represents the outer-coarsening which is the auto-correlation function in isothermal ageing (without perturbation $\tau_p = 0$). Thus, the memory correlation function equation (24) behaves non-monotonically with time in the healing stage and exhibits a peculiar 'memory peak' because of the two competing factors; $\rho_{\mathrm{rec}}(\tau_h)$ increases while $C_0(\tau_h + t_w, t_w)$ decreases with τ_h.

In figure 2 we show the memory auto-correlation function of the four-dimensional EA model after a bond perturbation of strength $p = 0.2$. By an independent numerical study of a bond-shift simulation performed in the same way as in a recent temperature-shift experiment [10, 25], we have checked that our time window lies almost entirely in the strongly perturbed regime with $p = 0.2$ as reported elsewhere [11]. In the scaling plot (b), the expected *multiplicative* recovery of bias (memory) (see equation (17)) is demonstrated.

In a previous study of this model [28] $\bar{\lambda} \sim 3.0$–3.5 was found by analysing the relaxation of thermo-remanent magnetization (TRM). As shown in the scaling plot (b) the present result appear to be consistent with $\lambda \sim 0.8$ and $\bar{\lambda} \sim 3.2$ (thus $\bar{\lambda}/\lambda \sim 4$) being consistent with the scaling relation equation (13) and the inequality $\bar{\lambda}/\lambda \geqslant 1$ equation (15). Indeed, it can be seen that the recovery time at which the data merge with the reference data of $C_0(\tau_h + t_w, t_w)$ (thus $\rho^A \to 1$) is already as large as $O(10^4)$ (MCS) even with very short perturbation $\tau_p = 10$ (MCS).

The factorization in equation (24) strongly suggests independence of the evolution of the amplitude of the bias or the order parameter and size of the domain. Consequently, somewhat surprisingly, the above result suggests that the memory peaks can be *always* identified no matter how long the perturbation is kept on. Note that nothing special happens when $L(\tau_p)$ exceeds $L(t_w)$. Only the amplitude of the signal will be smaller for longer perturbation so that higher resolution is required.

Rather amusingly, the factorization equation (24) allows us to extract the growth of the amplitude of the bias ρ even with no knowledge of the equilibrium state $\Gamma/\bar{\Gamma}$ by using the auto-correlation functions which are gauge-invariant quantities. Probably it is interesting to apply the same trick to other spin-glass models. In the $d = 3$ Ising EA model the data reported in [31] on the relaxation of the auto-correlation function in isothermal ageing suggest roughly $\bar{\lambda} \sim 2$ and hence $\bar{\lambda}/\lambda \sim 2$ assuming the scaling relation equation (13).

Weakly perturbed regime. Naturally the factorization of the time evolution of the bias and the size of the domains equation (24) should also hold in the weakly perturbed regime. In the four-dimensional EA model we also performed bond cycling simulations operated in the weakly perturbed regime with very small p such as $p = 0.02$. There, we found that the recovery of the bias is *additive* as suggested by equation (21) and the recovery time was found to be the trivial one $\tau_{\mathrm{rec}} \sim \tau_p$ being consistent with equation (22).

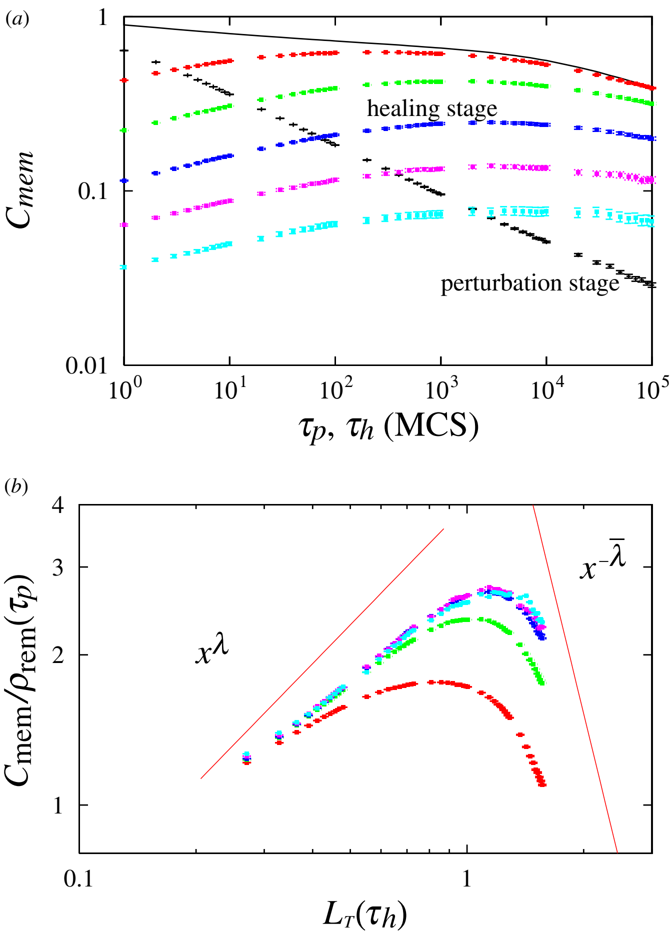

Figure 2. Memory correlation function of the four-dimensional EA model. The perturbation is a bond perturbation of strength $p = 0.2$. The temperature is $T = 1.2$ ($T_g = 2.0$). The system size is $N = 24^4$ which is large enough to avoid finite size effects within the present time window [28]. The initial waiting time is fixed as $t_w = 10^4$ (MCS). (*a*) The data points labelled 'perturbation stage' are $C_{\text{mem}}(\tau_p + t_w, t_w)$. Other data points are those in the healing stage $C_{\text{mem}}(\tau_h + \tau_p + t_w, t_w)$ after various durations of the perturbation $\tau_p = 10, 10^2, 10^3, 10^4, 10^5$ (MCS) from the top to the bottom. Note that the last is even larger than the initial waiting time t_w. The solid line is the reference curve of $C_0(\tau_h + t_w, t_w)$ obtained by a simulation of isothermal ageing with $t_w = 10^4$ (MCS). (*b*) The memory correlation functions scaled by the *remanent bias* $\rho_{\text{rem}}(\tau_p)$ are shown. The remanent bias is directly read off as $\rho_{\text{rem}}(\tau_p) = C_0(\tau_p + t_w, t_w)$. Here the dynamical length $L_{T=1.2}(t)$ is used, which has been obtained in a previous study of the same model [28, 30]. The straight lines are the power laws x^{λ} and $x^{-\bar{\lambda}}$ with $\lambda = 0.8$ and $\bar{\lambda} = 3.2$.

5.2. Magnetic susceptibilities

In experiments ac/dc magnetic susceptibilities are often used to study dynamics of spin-glass materials. As noted in section 3, these are also essentially gauge-invariant quantities. Here we discuss possible scaling properties of those in the healing stage.

The relaxation of the ac susceptibility of frequency ω can be considered as a probe of the increase of the effective stiffness of a droplet excitation of size $L_T(1/\omega)$ due to the decrease of domain wall density [26]. The scaling ansatz which follows this picture has been supported by recent numerical and experimental studies [28, 32–34]. In the healing stage, there must be excess contributions from the domain walls around the islands of the minority phase. A given spin can be surrounded by such a wall with probability $p_{\mathrm{minor}}(\tau_h, \tau_p) = (1-\rho_{\mathrm{rec}}(\tau_h, \rho_{\mathrm{rem}}(\tau_p)))$ (see equation (5) which increases with τ_p and decreases with τ_h. Then a natural scaling is

$$\chi''(\omega, \tau_h + \tau_p + t_w) = p_{\mathrm{minor}}(\tau_h; \tau_p)\chi_0''(\omega, \tau_h) + \chi_0''(\omega, \tau_h + t_w) \qquad \text{for} \qquad p_{\mathrm{minor}} \ll 1 \tag{25}$$

Here $\chi_0''(\omega, t)$ is the ac susceptibility of isothermal ageing starting from random initial condition at $t = 0$. The first term is the excess response due to the minority phase. Since $p_{\mathrm{minor}}(\tau_h, \tau_p)$ decreases with time τ_h, the excess part slowly fades away. The second term on the right-hand side is due to the outer-coarsening which is just the ac susceptibility without perturbation. If the cycling is operated in the weakly perturbed regime, the excess part will fade away at the timescale $\tau_{\mathrm{rec}}^{\mathrm{weak}}$ given in equation (22) while it will take an extremely long time $\tau_{\mathrm{rec}}^{\mathrm{strong}}$ given in equation (18) in the strongly perturbed regime. It is interesting to note that an anomalously large recovery time that apparently exceeds the simple estimate equation (22), which is valid only in the perturbative regime, has been found in recent measurements of the ac susceptibility [11, 24].

In isothermal ageing which starts from a random initial condition at $t = 0$ the zero-field-cooled (ZFC) susceptibility $M_{\mathrm{ZFC}}(\tau = t - t_w)$ measured under a probing field switched on after waiting time t_w exhibits a rapid increase at around $\tau \sim t_w$. The latter is reflected as a peak of the relaxation rate $S(\tau) = \mathrm{d}S(\tau)/d\log(\tau)$ at around $\tau \sim t_w$ [35]. In the cycling operated in the strongly perturbed regime, it is very likely to happen that the population of the minority phase within the ghost domains $p_{\mathrm{minor}}(\tau_h)$ remains non-zero at the timescale $\tau_h \sim t_w$. This may explain the substantial reduction of the amplitude of the memory peak of $S(\tau_h)$ at around $\tau_h \sim t_w$ in one-step temperature-cycling experiments operated in the strongly perturbed regime [11, 36].

6. Renormzalization of slow switching effects

So far we have considered idealized situations that perturbations are switched on/off instantaneously, which is not possible in reality. For example, typical heating/cooling rates are $v_T = 1$ K s^{-1} in 'quench' experiments [9], which is equivalent to $v_T = 10^{-15}$ J/MCS in simulations (assuming $T_g = 10$ K and the microscopic timescale $\tau_0 = 10^{-13}$ s). The surprising weakness of the heating/cooling rate effect in spin-glasses [9] already suggests the relevance of the chaos effects.

Let us illustrate here some important consequences of such a slow switching by considering a continuous bond change protocol as an example. Suppose that the signs of a fraction p of $\pm J$ bonds in a temporally set $\mathcal{J}(t)$ are changed randomly in a unit time τ_0. After some transients, the system should become stationary by the chaos effect such that the size of the ghost domains of $\Gamma^{\mathcal{J}(t)}$ becomes constant in time L_{v_J}, which decreases by increasing the bond change rate $v_J = pJ/\tau_0$. Then we can consider, for example, a one-step bond cycling $J_A(t_w) \to J_B(\tau_p) \to J_A(\tau_h)$ with such a gradual bond changes. The point is that at length scales smaller than L_{v_J} the whole cycling process just amounts to successive operations in the weakly perturbed regime. There the multiplicative effects equation (19) are avoided as explained in section 3.1. Then the cycling can be coarse-grained by taking L_{v_J} as the new microscopic length scale instead of the overlap length $\xi(\delta)$ between A and B, which yields a coarse-grained cycling $J_A \to J_B \to J_A$ operated in a strongly perturbed regime with instantaneous bond changes. The scaling properties of the strongly perturbed regime will hold for the latter but the original overlap length $\xi(\delta)$ should be replaced by the *renormalized* *overlap length* L_{v_J}, for example in equation (18), which leads to a certain 'rounding' of the strong chaos effects in realistic circumstances.

Although the temperature dependence of the growth law equation (7) induces some obvious complications, essentially the same argument for the case of continuous temperature changes leads to a corresponding renormalized overlap length L_{v_T} which decreases with increasing heating/cooling rate v_T.

7. Conclusion

To summarize we have studied how spin-glasses heal after being exposed to strong perturbations which induce the chaos effects in simple perturbation–healing protocols (e.g. one-step temperature-cycling). The bias or the order parameter within the ghost domains decays as $L^{-\bar{\lambda}}$ in the perturbation stage and increases as L^{λ} in the healing stage with increasing dynamical length scales L. The inequality of the exponents $\bar{\lambda} \geqslant \lambda$ immediately suggests anomalously large recovery times of the order parameter. The memory auto-correlation function is suited for direct examination of the time evolution of the order parameter. It should exhibit the memory peak in the healing stage at the timescale of the 'age' imprinted in the ghost domains due to the parallel evolution of the order parameter (inner-coarsening) and the size of the ghost domains (outer-coarsening). The predictions were checked quantitatively by numerical simulations in the four-dimensional EA model. It should be very interesting to measure experimentally the memory auto-correlation function by the noise-measurement technique [27], for example, in the standard one-step temperature-cycling protocol. Extensive experimental and numerical investigations which examine the ghost domain scenario in other observables such as the ac/dc magnetic susceptibilities will be reported elsewhere [11]. Important features of the weakly perturbed regime of the chaos effect were also discussed, which leads to a proposal to take into account the effect of finiteness of switching on/off the perturbations in experimental circumstances by the renormalized overlap length.

Acknowledgments

The author thanks Jean Philippe Bouchaud, Koji Hukushima, Petra E Jönsson, Anaël Lemaître, Philipp Maass, Roland Mathieu, Per Nordblad, Marta Sales, Falk Scheffler and Hajime Takayama for collaborations related to this work and useful discussions.

References

[1] Bray A J and Moore M A 1987 *Phys. Rev. Lett.* **58** 57
[2] Fisher D S and Huse D A 1988 *Phys. Rev.* B **38** 386
[3] Fisher D S and Huse D A 1991 *Phys. Rev.* B **43** 10728
[4] Banavar J R and Bray A J 1987 *Phys. Rev.* B **35** 8888
[5] Ney-Nifle M and Hilhorst H J 1993 *Physica* A **193** 48
[6] Aspelmeier T, Bray A J and Moore M A 2002 *Phys. Rev. Lett.* **89** 1972021
[7] Rizzo T and Crisnti A 2003 *Phys. Rev. Lett.* **90** 137201
[8] Sales M and Yoshino H 2002 *Phys. Rev.* E **65** 066131
[9] Jonason K, Vincent E, Hammann J, Bouchaud J-P and Nordblad P 1998 *Phys. Rev. Lett.* **81** 3243
[10] Jönsson P, Yoshino H and Nordblad P 2002 *Phys. Rev. Lett.* **89** 097201
[11] Jönsson P E, Mathieu R, Nordblad P, Yoshino H, Katori H A and Ito A 2003 *Preprint* cond-mat/0307640
[12] Yoshino H, Lemaître A and Bouchaud J-P 2001 *Eur. Phys. J.* B **20** 367
[13] Scheffler F, Yoshino H and Maass P 2003 *Phys. Rev.* B **68** 060404R
[14] Sasaki M and Martin O 2003 *Phys. Rev. Lett.* **91** 097201

[15] Kurchan J 1997 Spin glasses: experimental facts, theoretical concepts, and open questions *Proc. 'Jamming and Rheology: Constrained Dynamics on Microscopic and Macroscopic Scales' (ITP, Santa Barbara) Preprint* cond-mat/9812347
[16] Bray A J 1994 *Adv. Phys.* **43** 357
[17] Bray A J and Kissner J G 1992 *J. Phys. A: Math. Gen.* **25** 31
[18] lefloch F, Hammann J, Ocio M and Vincent E 1992 *Europhys. Lett.* **18** 647
[19] Andersson J O, Mattsson J and Nordblad P 1993 *Phys. Rev.* B **48** 13977
[20] Papadopoulou E L, Nordblad P, Svedlindh P, Schöneberger R and Gross R 1999 *Phys. Rev. Lett.* **82** 173
[21] García-Palacios J L, Jönsson P and Svedlindh P 2000 *Phys. Rev.* B **61** 6726
[22] Leheny R L and Nagel S R 1998 *Phys. Rev.* B **57** 5154
[23] Bellon L, Ciliberto S and Laroche C 2000 *Europhys. Lett.* **51** 551
[24] Sasaki M, Dupis V, Bouchaud J-P and Vincent E 2002 *Eur. Phys. J.* B **29** 469
[25] Jönsson P, Yoshino H and Nordblad P 2003 *Phys. Rev. Lett.* **90** 059702
[26] Fisher D S and Huse D A 1988 *Phys. Rev.* B **38** 373
[27] Hérisson D and Ocio M 2002 *Phys. Rev. Lett.* **88** 257202
[28] Yoshino H, Hukushima K and Takayama H 2002 *Phys. Rev.* B **66** 064431
[29] Koper G J M and Hilhorst H J 1988 *J. Phys. (Paris)* **49** 429
[30] Hukushima K, Yoshino H and Takayama H 2000 *Prog. Theor. Phys.* **138** 568
[31] Kisker M S J, Santen L and Rieger H 1996 *Phys. Rev.* B **53** 6418
[32] Schins A G *et al* 1993 *Phys. Rev.* B **48** 16524
[33] Komori T, Yoshino H and Takayama H 2000 *J. Phys. Soc. Japan* **69** 1192
[34] Jönsson P E, Yoshino H, Nordblad P, Katori H A and Ito A 2002 *Phys. Rev. Lett.* **88** 257204
[35] Lundgren L, Svedlindh P, Nordblad P and Beckman O 1983 *Phys. Rev. Lett.* **51** 911
[36] Granberg P, Lundgren L and Nordblad P 1990 *J. Magn. Magn. Mater.* **92** 228